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SOME ASPECTS OF BARODESY

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Summary. Barodesy is a mathematical frame for constitutive modelling that makes possible to derive a constitutive relation from basic properties of soil. As such are considered those ones that are asymptotically obtained with monotonous deformation. The latter erase any initial disturbances imposed erratically by the sample preparation. The obtained equations have a simple mathematical structure which facilitates understanding of the underlying processes and numerical implementation. In this way, barodesy could serve as a basis for further developments in a field with many open questions and which is erroneously considered as completed.

1 DIRECTIONS OF PROPORTIONAL PATHS

GOLDSCHEIDER observed that proportional strain paths starting from the stress-free state lead to proportional stress paths. A proportional strain path is characterized by a constant stretching, $\mathbf{D} = \text{const}$, hence the normalized stretching $\mathbf{D}^0 := \mathbf{D}/|\mathbf{D}|$ indicates its direction. The direction of the pertinent proportional stress path is given by a tensor $\mathbf{R}$. We seek a relation $\mathbf{R}(\mathbf{D}^0)$. We know that all contractant proportional strain paths should produce proportional stress paths that lie within the compression octant of the principal stress space. This means that all principal values of $\mathbf{R}$ must be negative if $\text{tr}\mathbf{D} < 0$. The condition $\det\mathbf{R} < 0$ iff $\text{tr}\mathbf{D}^0 < 0$ suggests to use the following function for the relation $\mathbf{R}(\mathbf{D}^0)$:

$$\mathbf{R}(\mathbf{D}^0) = -\exp(a\mathbf{D}^0) \tag{1}$$

The collection $\mathscr{E}$ of all proportional strain paths with $\text{tr}\mathbf{D}^0 = 0$ consists of volume preserving ('undrained') proportional strain paths and defines the limit between dilatant and contractant paths. The relation (1) maps $\mathscr{E}$ into the collection $\mathscr{S}$ of the corresponding proportional stress paths. Clearly, $\mathscr{S}$ is the critical state surface in stress space. Interestingly, the cross section of $\mathscr{S}$, as obtained with equ. (1), with a deviatoric plane $\text{tr}\mathbf{T} = \text{const}$ is a curve that practically concides with the MATSUOKA-NAKAI limit curve.

With the abbreviations $\sigma := |\mathbf{T}|$ and $\dot{\varepsilon} := |\mathbf{D}|$ a constitutive equation of the rate type can be formulated that holds for proportional strain paths: $\dot{\mathbf{T}} = h\mathbf{R}^0\dot{\varepsilon}$. Herein, h is a function of σ and expresses the fact that the stiffness increases with increasing σ.

2 ASYMPTOTIC PATHS

GOLDSCHEIDER observed also that starting from a non-vanishing stress state and applying a proportional strain path leads asymptotically to the proportional stress path that would be obtained starting from the stress-free state. This behaviour can be modelled if we add to the aforementioned constitutive equation a second term which is proportional to the stress $\mathbf{T}$. This leads to the following final form of the barodetic constitutive equation:

$$\dot{\mathbf{T}} = h(f\mathbf{R}^0 + g\mathbf{T}^0)\dot{\varepsilon}, \tag{2}$$

which allows to describe many effects of soil behaviour in an elegant way. Of course, to accomplish the equation, the scalar quantities f and g have to be specified.

Limit states, defined by vanishing stiffness $\dot{\mathbf{T}} = \mathbf{0}$, are manifested either as peak or residual (critical) limit states. In barodesy, $\dot{\mathbf{T}} = \mathbf{0}$ implies $f\mathbf{R}^0 + g\mathbf{T}^0 = \mathbf{0}$. This tensorial equation implies the 'flow rule' $\mathbf{R}^0 = \mathbf{T}^0$ and the 'yield condition' $f + g = 0$. Critical limit states are obtained with $\delta := \mathrm{tr}\,\mathbf{D}^0 = 0$ and $e = e_c$, whereas peak limit states are obtained with $\delta > 0$ and $e < e_c$. A simple way to model limit states of either type is to set $f + g = \delta + c_3(e_c - e)$ with e_c being the critical void ratio. It can be shown that the critical state line, i.e. a relation $e_c(\sigma)$ can be derived from the above formalism.

3 LIMIT CYCLES

Proportional paths are not the only attractors in barodesy. Being an ordinary differential equation, it exhibits also limit cycles or cyclic orbits as further attractors. In mechanics, the existence of such attractors is known as 'shake-down' and implies that stress cycles lead asymptotically to cyclic changes of void ratio. For oedometric deformation, this implies also cyclic strain. This phenomenon is related to vibrocompaction. However, the original barodetic equation needs some modification to realistically model cyclic loading and small strain stiffness. This modification can be achieved by incorporating the acceleration of deformation or, more precisely, the time rate of normalized stretching.

4 BENEFITS OF BARODESY

As hypoplasticity, barodesy avoids any whatsoever defined surfaces in stress space to describe the material behaviour. Even if some surfaces can be a posteriori derived from the original equations, they are not necessary for the calculation, and this is an advantage, as it renders any return mapping superfluous. In addition, barodesy constitutes a frame of amazing simplicity to describe material behaviour including barotropy and pyknotropy. And last but not least, it offers a simple theoretical platform to understand and describe the behaviour of soil.